(1)

Linear Algebra

27/5/17

Gauss Jordan Elimination

Matrix (matrices)

- Procedure to find out solution of system of equation ⇒ Jordan Elimination

· matrix

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ / $B = \begin{pmatrix} \end{pmatrix}$.

$m \times n$

Denoted by Capitall letters and elements contained are small letters. represented by eg. $m \times n$. 2×2

m = rows, n = columns

· if we say $n \times n$ or $m \times m$ (rows = colums) ⇒ square matrix.

types of matrices

① Column vector

only one column. eg. $(1, 2, 3)$

$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

because number of columns is 1

$m \times 1$

3×1

② Row vector

only one Row.

$1 \times n$

1×1 satisfies both column vector and Row vector.

③ square matrix

when number of rows = no. of colums. ~~eg~~ $(m = n)$

④ Identity:

\ → Primary diagonal

/ → secondary diagonal.

Date:

- Identity matrix is a square matrix whose all ~~leading~~ ~~elements~~ diagonal elements are 1

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(5) Diagonal matrix:

$$\begin{bmatrix} 2 & y \\ x & 2 \end{bmatrix}$$

(diagonal elements are same and not zero)

(6) Scalar matrix:

$$\begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix}$$

(diagonal elements are same while others are zero)

Identity: is a square, diagonal and scalar matrix.

Diagonal: is a square, non scalar, diagonal; non identity

scalar: a is a square, non identity, non diagonal

- Identity has 1s in its diagonal, and non diagonal are zero. It is a square matrix. (diagonals are same.)

- Diagonal has non 1s in its diagonal and non diagonal are non zeros. it is a square matrix. (diagonals are same)
- Scalar has non 1s in its diagonal and non diagonals are zeros. it is a square matrix. (diagonals are same)

A, $m_A \times n_A$, B, $m_B \times n_B$ then

multiplying ~~A × B~~ A and B

if $m_A \times \underbrace{n_A \quad m_B}_{\text{equal}} \times n_B$

∴ order will be $m_A \times n_B$

- No. of rows of the first and no. of columns of the second are equal then only we can multiply two matrices

Scalar Addition	2 + A	~~(True)~~ (False)
Scalar Subtraction	A - 2 / 2 - A	(False)
Scalar Division	A/2	~~(True)~~ (False)
→ Scalar Multiplication	2 A	(True)

Transpose of a matrix

Change of rows to columns and columns to Rows.

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$$a_{ij} = \begin{cases} i+j & i=j \\ i-j & i\neq j \end{cases}$$

if $i=1$ and $j=1$

5

$$a_{ij} \quad \begin{array}{cc} a_{11} & a_{12} \\ \end{array}$$

$$\begin{Bmatrix} 2 & -1 \\ 1 & 4 \\ 2 & 1 \end{Bmatrix}$$

a_{21} a_{22}

a_{31} a_{32}

formulate A matrix whose elements are.

$$a_{ij} = \begin{cases} i+j & \text{if } i=j \\ i-j & \text{if } i\neq j \end{cases}$$

$$\begin{pmatrix} 2 \end{pmatrix}$$

formulate a 5×5 matrix whose elements are

$$a_{ij} = \begin{cases} \ln(i+j) & i = j \\ ie^{j} & i \neq j \end{cases}$$

$$a_{ij} = \begin{pmatrix} 0.69 & 7.38 & 20.08 & . & . \\ 5.43 & 1.386 & & & \\ 8.15 & 22.1 & 1.79 & & \\ 10.8 & 29.5 & & 2.07 & \\ 13.5 & 36.9 & & & 2.30 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 & -2 & 4 \\ 2 & -1 & 5 \\ -1 & 3 & -3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{pmatrix}$$

Solution

$$\begin{bmatrix} 1(1) - 2(0) + 4(0) & 1(0) - 2(1) + 4(0) & 1(2) - 2(-1) + 4(2) \\ 2(1) - 1(0) + 5(0) & 2(0) - 1(1) + 5(0) & 2(2) - 1(-1) + 5(2) \\ -1(1) + 3(0) - 3(0) & -1(0) + 3(1) - 3(0) & -1(2) + 3(-1) - 3(2) \end{bmatrix}$$

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$$\begin{pmatrix} 1 & -2 & 12 \\ 2 & -1 & 15 \\ -1 & 3 & -11 \end{pmatrix}$$

Gauss Jordan elimination
it is same with Gauss Jordan reduction method.

(1) $x_1 - 2x_2 + 4x_3 = 12$
$2x_1 - x_2 + 5x_3 = 18$
$-x_1 + 3x_2 - 2x_3 = -8$

coefficient   variable matrix

$A\vec{X} = B$

$$\left(\begin{array}{ccc|c} 1 & -2 & 4 & 12 \\ 2 & -1 & 5 & 18 \\ -1 & 3 & -2 & 8 \end{array}\right)$$

Augmented matrix.

in Gauss Jordan method, we reduce this matrix into identity by apply row operations. we do any scalar mul / div / Add / sub to transform this coefficient matrix into ~~identy~~) , identity).

soln.

best way to transform any coefficient to identy, we move diagonals and ~~tra~~ transform non diagonals to zero one by one.

$$\begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array} \left(\begin{array}{ccc|c} 1 & -2 & 4 & 12 \\ 2 & -1 & 5 & 18 \\ -1 & 3 & -2 & 8 \end{array}\right)$$

Add R_1 into R_3.

$$\left(\begin{array}{ccc|c} 1 & -2 & 4 & 12 \\ 2 & -1 & 5 & 18 \\ 0 & 1 & 2 & 4 \end{array}\right)$$

$R_2 - 2R_1$

$$\left(\begin{array}{ccc|c} 1 & -2 & 4 & 12 \\ 0 & 3 & -3 & -6 \\ 0 & 1 & 2 & 4 \end{array}\right)$$

multply R_2 by $1/3$

$$\left(\begin{array}{ccc|c} 1 & -2 & 4 & 12 \\ 0 & 1 & -1 & -3 \\ 0 & 1 & 2 & 4 \end{array}\right)$$

$R_3 - R_2$

$$\left(\begin{array}{ccc|c} 1 & -2 & 4 & 12 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 3 & 6 \end{array}\right)$$

$2R_2 + R_1$

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & 8 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 3 & 6 \end{array}\right)$$

multiply R_3 by $\frac{1}{3}$

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & 8 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

$R_3 + R_2$

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & 8 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

$R_1 - 2R_3$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

$x_1 = 4$, $x_3 = 2$
$x_2 = 0$

$(x_1, x_2, x_3) = (4, 0, 2)$ //

Question

Solve the system of equations by Gauss Jordan Elimination method.

$$x_1 + x_2 + x_3 = 2$$
$$2x_1 + 3x_2 + x_3 = 3$$
$$x_1 - x_2 - 2x_3 = -6$$

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 3 & 1 & 3 \\ 1 & -1 & -2 & -6 \end{array}\right]$$

$R_3 - R_1$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 3 & 1 & 3 \\ 0 & -2 & -3 & -8 \end{array}\right)$$

$R_2 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & -2 & -3 & -8 \end{array}\right]$$

$\cancel{2R_1 + R_3}$

$$\cancel{\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 2 & 0 & -1 & -4 \end{array}\right]}$$

$R_1 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & -1 & -1 \\ 0 & -2 & -3 & -8 \end{array}\right]$$

$2R_2 + R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & -10 \end{array}\right]$$

$R_2 + R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & 0 & -11 \\ 0 & 0 & 1 & -10 \end{array}\right]$$

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$x_1 = -1$

$x_2 = 1$

$x_3 = 2$

$\underline{R_1 - 2R_3}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -17 \\ 0 & 1 & 0 & -11 \\ 0 & 0 & 1 & -10 \end{array}\right]$$

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Determinant of a matrix

1×1 matrix

$A = (7)$

$|A| = 7.$

Determinant of a 1×1 matrix is the element itself

2×2 matrix

$A = \begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix} = (2\times1)-(4\times3)$

$= 2-12$

$= -10$

$|A| = \begin{vmatrix} 2 & 3 \\ 4 & 1 \end{vmatrix} = -10$

3×3 matrix

$A = \begin{pmatrix} 1 & 0 & -2 \\ 3 & 5 & 3 \\ 2 & 1 & 2 \end{pmatrix}$

$|A| = \begin{vmatrix} 1 & 0 & -2 \\ 3 & 5 & 3 \\ 2 & 1 & 2 \end{vmatrix} = 1\begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} - 0(+2)\begin{vmatrix} 3 & 5 \\ 2 & 1 \end{vmatrix}$

$|A| = 21$

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Minor of the element a_{ij}

It is denoted by M_{ij} and is the determinant of the matrix that remains after deleting row i and column j of A.

Cofactor of the element a_{ij}

It is denoted by C_{ij} and is given by

$$C_{ij} = (-1)^{i+j} M_{ij}$$

Note: the minor and cofactor differs in at most sign. $C_{ij} = \pm M_{ij}$

i stands for row and j stands for column.

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 4 & -1 & 2 \\ 0 & -2 & 1 \end{pmatrix}$$

Minor of a_{11} : $M_{11} = \begin{vmatrix} -1 & 2 \\ -2 & 1 \end{vmatrix} = (-1) - (-4)$

$-1 + 4$

$M_{11} = 3$

cofactor of a_{11} : $C_{11} = (-1)^{1+1} \cdot 3 = (1-)^2 \times 3$

$C_{11} = 3$

$C_{ij} = M_{ij}$

Date

$$A_c = \begin{vmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{vmatrix} \qquad \begin{vmatrix} 1 & 0 & 3 \\ 4 & -1 & 2 \\ 0 & -2 & 1 \end{vmatrix}$$

$$c_{11} = (-1)^2 \cdot M_{11} \begin{vmatrix} 2 \\ -2 & 1 \end{vmatrix}$$

$$A_{\psi} = \begin{vmatrix} 3 & 2 & -8 \\ 6 & -2 & -2 \\ 3 & -10 & -1 \end{vmatrix}$$

$c_{11} = 3$

$c_{12} = -4$

$c_{13} = -8$

$c_{21} = -6$ cofact

$c_{22} = 1$

$c_{23} = 2$

$c_{31} = 3$

$c_{32} = 10$

$c_{33} = -1$

The determinant of a square matrix is the sum of the products of the elements of the first row and their cofactors.

if A is 3×3, $|A| = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$

if A is 4×4, $|A| = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13} + a_{14}C_{14}$

$\vdots$

if A is $n \times n$, $|A| = a_{11}C_{11} + a_{12}C_{12} + \dots + a_{1n}C_{1n}$

Inverse of a matrix

$\frac{1}{A}$, $A^{-1} = \frac{1}{|A|} \times$ adjoint of A $\quad \frac{1}{A}$

$= \frac{1}{|A|} A_j \qquad A^{-1}$

A_j of 2×2

$$A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} \qquad A_j = \begin{pmatrix} 4 & -3 \\ -1 & 2 \end{pmatrix}$$

A_j of 3×3

$$A_j = (A_C)^t$$

$\therefore$ Inverse of 3×3 $\quad A^{-1} = \frac{1}{|A|} \times (A_C)^t$

(b)

a_{11} a_{12} a_{13}
a_{21} a_{22} a_{23}
a_{31} a_{32} a_{33}

$$A = \begin{vmatrix} 2 & 0 & 3 \\ -1 & 4 & -2 \\ 1 & -3 & 5 \end{vmatrix} \qquad A_M = \begin{vmatrix} 14 & -3 & -1 \\ 9 & 7 & -6 \\ -12 & -1 & 8 \end{vmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times A_j$$

$$\frac{1}{|A|} \times (A_c)^t$$

$C_{11} = (-1)^2 \times 14 = 14$

$C_{12} = (-1)^3 \times -3 = 3$

$C_{13} = (-1)^4 \times -1 = -1$

$C_{21} = (-1)^3 \times 9 = -9$

$C_{22} = (-1)^4 \times 7 = 7$

$C_{23} = (-1)^5 \times -6 = 6$

$C_{31} = (-1)^4 \times -12 = -12$

$C_{32} = (-1)^5 \times -1 = -1$

$C_{33} = (-1)^6 \times 8 = 8$

$$A_c = \begin{vmatrix} 14 & 3 & -1 \\ -9 & 7 & 6 \\ -12 & -1 & 8 \end{vmatrix} \qquad (A_c)^T = \begin{vmatrix} 14 & 9 & -12 \\ 3 & 7 & -1 \\ -1 & 6 & 8 \end{vmatrix}$$

$|A| = + - +$

$|A| = (2 \times 14 - 0 + 3 \times -1) = 25$

$$A^{-1} = \frac{\begin{vmatrix} 14 & 9 & -12 \\ 3 & 7 & -1 \\ -1 & 6 & 8 \end{vmatrix}}{25} = \begin{vmatrix} 14/25 & 9/25 & -12/25 \\ 3/25 & 7/25 & -1/25 \\ -1/25 & 6/25 & 8/25 \end{vmatrix}$$

• ~~All~~ the matrices whose determinants are <u>zeros</u> are <u>singular</u> and their <u>inverse is not possible</u>. So inverse exits of only <u>non singular matrices</u>

1. Solve the system of equation using Inverse of a matrix

$$x_1 + 3x_2 + x_3 = -2$$
$$2x_1 + 5x_2 + x_3 = -5$$
$$x_1 + 2x_2 + 3x_3 = 6$$

$$\begin{pmatrix} 1 & 3 & 1 \\ 2 & 5 & 1 \\ 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -2 \\ -5 \\ 6 \end{pmatrix}$$

$$A \quad X = B$$

$$AX = B \quad - (1)$$
$$X = A^{-1}B \quad - (2)$$

$$A^{-1} = (A_C)^t \times \frac{1}{|A|}$$

$$A_M = \begin{vmatrix} 13 & 5 & -1 \\ 7 & 2 & -1 \\ -2 & -1 & -1 \end{vmatrix} \qquad A_C = \begin{vmatrix} -13 & 5 & 1 \\ 7 & -2 & -1 \\ 2 & -1 & 1 \end{vmatrix}$$

a_{11} a_{12} a_{13}
a_{21} a_{22} a_{23}
a_{31} a_{32} a_{33}

$$(A_C)^t = \begin{vmatrix} -13 & 7 & 2 \\ 5 & -2 & -1 \\ 1 & -1 & 1 \end{vmatrix}$$

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$$|A| = 1(1 \times 13) - (3 \times 5) + \times -1$$

$$|A| = 13 - 15 \quad = -3$$

$$|A| = -3$$

$$A^{-1} = \frac{1}{|A|} \times (AC)^t$$

$$A^{-1} = \begin{pmatrix} -13/-3 & 7/-3 & 2/-3 \\ 5/-3 & -2/-3 & -1/-3 \\ 1/-3 & -1/-3 & 1/-3 \end{pmatrix} \times \begin{pmatrix} -2 \\ -5 \\ 6 \end{pmatrix}$$

$$X = A^{-1} \times B$$

$$x = \begin{vmatrix} -13/-3 \times -2 & 7/-3 \times -5 & 2/-3 \times 6 \\ 5/-3 \times -2 & -2/-3 \times -5 & -1/-3 \times 6 \\ 1/-3 \times -2 & -1/-3 \times -5 & 1/-3 \times 6 \end{vmatrix} = \begin{vmatrix} -26/3 & 35/3 & -4 \\ 10/3 & -10/3 & 2 \\ 2/3 & -5/3 & -2 \end{vmatrix}$$

$$\begin{vmatrix} -1 \\ \\ \end{vmatrix}$$

Invertible Matrices

Solve the system of equations by using the inverse

Crammer's Rule

let $Ax = B$ be a system of linear equations in ... such that $|A| \neq 0$. the system has a unique solution given by

$$x_1 = \frac{|A_1|}{A}, \quad x_2 = \frac{|A_2|}{|A|}, \ldots, x_i = \frac{|A_i|}{|A|}$$

where A_i is the matrix obtained by replacing column i of A with B

solve this system

$$x_1 + 3x_2 + x_3 = -2$$
$$2x_1 + 5x_2 + x_3 = -5$$
$$x_1 + 2x_2 + 3x_3 = 6$$

$$x_1 = \frac{\begin{vmatrix} -2 & 3 & 1 \\ -5 & 5 & 1 \\ 6 & 2 & 3 \end{vmatrix} \Rightarrow |A_1|}{\begin{vmatrix} 1 & 3 & 1 \\ 2 & 5 & 1 \\ 1 & 2 & 3 \end{vmatrix} \Rightarrow |A|} = \frac{-3}{-3} = 1$$

$$x_2 = \frac{\begin{vmatrix} 1 & -2 & 1 \\ 2 & -5 & 1 \\ 1 & 6 & 3 \end{vmatrix}}{-3} = \frac{26}{-3}$$

10/6/2017

(Q) A square matrix is called an upper triangular matrix if all the elements below the main diagonal are zero. it is called a lower triangular matrix if all the elements above the main diagonal are zero.

example

$$\begin{bmatrix} 3 & 8 & 2 \\ 0 & 1 & 5 \\ 0 & 0 & 9 \end{bmatrix} \qquad \begin{bmatrix} 7 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 9 & 8 \end{bmatrix}$$

upper triangle   lower triangle

Theorem:- The determinant of a triangular matrix is the product of its diagonal elements.

Evaluate the determinant of

(i) $\begin{bmatrix} 1 & 0 & 2 & 1 \\ 2 & -1 & 1 & 0 \\ 1 & 0 & 0 & 3 \\ -1 & 0 & 2 & 1 \end{bmatrix}$ upper

(ii) $\begin{bmatrix} 1 & -1 & 0 & 2 \\ -1 & 1 & 2 & 3 \\ 2 & -2 & 3 & 4 \\ 6 & -6 & 5 & 1 \end{bmatrix}$ lower.

Solution for (i)

$R_1 + R_4$

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 2 & -1 & 1 & 0 \\ 1 & 0 & 0 & 3 \\ 0 & 0 & 4 & 2 \end{bmatrix}$$

$R_2 - 2R_1$

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & -1 & -3 & -2 \\ 1 & 0 & 0 & 3 \\ 0 & 0 & 4 & 2 \end{bmatrix}$$

$R_3 - R_1$

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & -1 & -3 & -2 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 4 & 2 \end{bmatrix}$$

$2R_3 + R_4$

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & -1 & -3 & -2 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

$6 \times -2 \times -1 \times 1$

Solution for (i)

(i)
$$\begin{bmatrix} 0 & 0 & 2 & 5 \\ -1 & 1 & 2 & 3 \\ 2 & -2 & 3 & 4 \\ 6 & -6 & 5 & 1 \end{bmatrix}$$

$R_2 - R_4$

(ii)
$$\begin{bmatrix} 0 & 0 & 2 & 5 \\ -1 & 1 & 0 & -2 \\ 2 & -2 & 3 & 4 \\ 6 & -6 & 5 & 1 \end{bmatrix}$$

$2R_2 + R_1$

(iii)
$$\begin{bmatrix} 0 & 1 & 2 & 5 \\ -1 & [illegible] & 0 & -2 \\ 8 & -8 & 2 & [illegible] \\ 6 & -6 & 5 & [illegible] \end{bmatrix}$$

$\longleftrightarrow \; ^{1}/R_3 - R_1$

(iv)
$$\begin{bmatrix} 0 & 0 \end{bmatrix}$$

(v)
$$\begin{bmatrix} \; \end{bmatrix}$$

(vi)

[illegible] will be easy

Equations Involving Determinants

① Solve for 'x' when

$$\begin{bmatrix} x & x+1 \\ -1 & x-2 \end{bmatrix} = 7$$

Singular matrix are those that has diagonals as zero.
non " " " " " " " " as nonzero
non " " " " " " are invertible, i.e. their inverses exists.

Soln

$$\{x(x-2) - (x+1)(-1)\} = 7$$

$$\{x^2 - 2x - (-x-1)\} = 7$$

$$x^2 - 2x + x + 1 = 7$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$x = 3 \; ; \quad x = -2$$

(1) Solve for x when matrix A is singular.

i) $\begin{vmatrix} x-1 & -2 \\ x-2 & x-1 \end{vmatrix}$ answer $x = \pm\sqrt{3}$

[illegible]

ii) $\begin{vmatrix} x & 0 & 2 \\ 2x & x-1 & 4 \\ -x & x-1 & x+1 \end{vmatrix}$ answer, $x = 0, 1, -3$

Property of determinants

Theorem: Let A be an $n \times n$ matrix and c be a non zero scalar.

(a) if a matrix B is obtained from A by multiplying the elements of a row (column) by c then $|B| = c|A|$.

(b) if a matrix B is obtained from A by interchanging two rows (or columns) then $|B| = -|A|$

(c) if a matrix B is obtained from A by adding a multiple of one row (column) to another row (or column) then $|B| = |A|$

Theorem: Let A be a square matrix. A is singular if:

(a) all the elements of the row (or column) are zero

(b) two rows (column) are equal

(c) two rows (columns) are proportional

Theorem: Let A and B be m×n matrices and c be a non zero
...lar

(a) Determinant of a scalar multiple : $|cA| = c^n |A|$

(b) Determinant of a product : $|AB| = |AB| = |A|\,|B|$

(c) Determinant of a transpose : $|A^t| = |A|$

(d) Determinant of an inverse : $|A^{-1}| = \dfrac{1}{|A|}$

... if A is a 2×2 matrix with $|A| = 4$. Use appropriate
...rem to complete the following determinants:

(a) $|3A|$

(b) $|A^2| = |A|\cdot|A|$

(c) $|5A^t A^{-1}| = 5^n |A^t|\,|A^{-1}| = 25 \times 4 \times$

(d) $|6AA^{-1}A^t|$

Inverse of a matrix by Gaussian Elimination

Step I: $(A/I) = \left(\begin{array}{ccc|ccc} 2 & 0 & 3 & 1 & 0 & 0 \\ -1 & 4 & -2 & 0 & 1 & 0 \\ 1 & -3 & 5 & 0 & 0 & 1 \end{array}\right)$

Augmented by 3×3 matrix

~~find the inverse of~~

Step II: Perform two operations

Step III: $(I/A^{-1}) = \left(\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & A^{-1} \\ 0 & 0 & 1 & \end{array}\right)$

Solution

$R_1 + R_2$

$$\begin{pmatrix} 1 & 4 & 1 \\ -1 & 4 & -2 \\ 1 & -3 & 5 \end{pmatrix}$$

$R_3 + R_2$

$$\begin{pmatrix} 1 & 4 & 1 \\ 0 & 1 & 3 \\ 1 & -3 & 5 \end{pmatrix}$$

$R_3 - R_1$

$$\begin{pmatrix} 1 & 4 & 1 \\ 0 & 1 & 3 \\ 0 & -3 & 4 \end{pmatrix}$$